

Rank-eleven multiplicity-one fusion rings

Exclusions, coherent models, and exact verification

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1 Result and mathematical scope

The input consists of 182 physical multiplication-table lines. All 182 satisfy the fusion-ring axioms, and exact invariant refinement separates every pair. There are 153 commutative and 29 noncommutative rings. Identifiers `R11_1_i` always retain physical input line i ; no sorting or basis change is allowed to redefine an identifier.

Status	Rings
Excluded over $\mathbb{C}$	155
Unitary realization verified	27
Unresolved after recorded attempts	0

An exclusion proves that no complex fusion category has the indicated based ring. A model proves existence of a unitary category, including coherence of the construction and an exact identification of all $11^3 = 1331$ coefficients. Neither assertion classifies all possible associators of a realized ring. No timeout, unsuccessful model search, floating-point test, or failed relaxation is used to assign an exclusion.

AI-generated research · independent verification pending. The proofs, code and data were developed with AI assistance. Independent human verification and Lean formalization remain pending. Exact automated checks establish only their stated scope.

Theorem 1 (Classification of the supplied multiplicity-one census). *Of the 182 supplied rank-eleven multiplicity-one fusion rings, exactly 27 admit a fusion categorification over $\mathbb{C}$. Each admits a unitary realization. The remaining 155 admit no complex fusion categorification.*

The theorem concerns based-ring isomorphism classes in the supplied dataset. Enumeration completeness is producer-reported: every duality stratum completed and the complete source is supplied, while independent integration checks establish tensor validity and uniqueness. The full enumeration was not independently repeated. The census provenance is retained separately in <https://github.com/sebastienpalcoux/fusion-ring-census>. The canonical manuscript and per-input classification evidence are in <https://github.com/sebastienpalcoux/grothendieck-ring-classification>.

2 Known criteria and the order of the audit

The audit first reused the literature and accepted lower-rank project methods: formal-codegree d -numbers, character-trace cyclotomicity, primitive codegrees, integral center dimensions, all fusion-subring indices, near-group and Tambara–Yamagami restrictions, zero/one-spectrum obstructions, cyclic isotypic integrality, module matrices, and inherited solvability arguments. It then applied complete signed-sphericalization and scalar-character checks before the center-dimension and reciprocity refinements. Model searches used established families before constructing additional coherent actions.

The detailed criterion-by-criterion inventory is `work/rank11/criteria_audit/CRITERIA_AUDIT.md`. It records hypotheses, sources, implementation boundaries, and conditions which are only unitary or braided. It covers the surveyed literature and the accepted project corpus; it does not claim that every conceivable theorem or consequence of the literature has been algorithmically decided.

The exact arithmetic audit checks 919 proper nontrivial fusion subrings, 286 distinct irreducible field polynomials, and the irreducible algebra-block decomposition of all 29 noncommutative inputs. Cyclotomicity is checked on character traces in the noncommutative case. The four arithmetic receipts give 97 excluded inputs. The cyclic test gives 66 exclusions with overlap. The zero/one-spectrum and module proofs use actual algebra and pentagon constraints, with independent integer replays. Overlapping proofs are retained in the ledger and are counted only once.

The signed analysis covers all 59 nonintegral inputs surviving the initial arithmetic and cyclic tests. It checks every admissible sign branch and every possible spherical dimension character. In particular, the six mixed dimension-40 branches are handled directly; they are not assumed pseudo-unitary. The dimension-lattice and two-character arguments below use that reduction as an explicit proof dependency.

For the final integral pair, all primary positivity criteria and the noncommutative Schur-product criterion pass by exact matrix identities. Nonnegative rational coupling certificates establish all of the tested Minkowski inequalities for every real $p \geq 1$ and every nonnegative input vector. These are genuine all-parameter passes, not numerical searches for witnesses. The new graded-extension argument below resolves the pair. The independent Titsworth–Hagge data crosswalk also checks their rank-eleven dimension-32 tensor: it is exactly input 30, already realized here.

2.1 Evidence and reproducibility

`work/rank11/ledger.json` is the per-input record of status, original tensor hashes, proof files, certificates, and replay commands. `work/rank11/ledger.csv` is its compact human-readable counterpart. The input hash and mathematical assertions are bound to the original tensor, not to approximate dimensions or an isomorphism signature alone. Run the portable replay from the extracted package root:

```
python work/rank11/replay_all.py
```

The accompanying README documents supported options, dependencies and the recorded complete replay. Discovery programs are kept distinct from accepted certificate checks. Checks reject mismatches explicitly even under `python -O`. The model audit independently reconstructs the highest-risk coherence equations and fusion coefficients.

3 Verified models

Line	One verified realization
1	$\text{Rep}(D_{32})$
10	$\text{TY}(C_{12})^{C_2}$, positive parity
24	$\text{SU}(2)_6^{C_2}$, odd parity coherence
26	$\text{SU}(2)_{10}$
28	$\text{PSU}(2)_{20}$
30	$\text{SO}(8)_2$
37	$\text{SO}(15)_2$
45	$\text{Rep}(D_{38})$
46	$\text{PSU}(2)_{21}$
48	$\text{Vec}_{C_{16}}^{\omega_1}$, equivariantized by $a \mapsto 15a$
57	V_4 zest of $\text{SO}(8)_2$, parameters $(0, 0, 0, 2)$
64	$\text{TY}(C_3 \times C_2^2)^{C_2}$, positive parity
68	$\text{SU}(2)_6^{C_2}$, trivial parity coherence
72	$\text{TY}(C_{12})^{C_2}$, negative parity
75	$\text{TY}(C_{15})^{C_2}$, negative parity
103	C_2 zest of $\text{SU}(2)_{10}$
104	$\mathcal{C}(G_{32}, \omega, V_4, 1)$, nonsplit Jordan construction
105	V_4 zest of $\text{SO}(8)_2$, parameters $(0, 0, 0, 1)$
112	$\text{Vec}_{C_{16}}^{C_2}$, action $a \mapsto 7a$
113	$\text{TY}(C_3 \times C_2^2)^{C_2}$, negative parity
146	$\text{Vec}_{C_{16}}^{\omega_1}$, equivariantized by $a \mapsto 7a$
150	V_4 zest of $\text{SO}(8)_2$, parameters $(2, 0, 0, 3)$
157	$\text{TY}(C_3^2)^{C_2}$, positive parity
171	$\text{TY}(C_{10})$
176	$\text{TY}(C_3^2)^{C_2}$, negative parity
179	Izumi Q_8 near-group category, equivariantized by C_3
182	$\text{Vec}_{C_{11}}$

Here D_m denotes a dihedral group of order m . Parameters, exact phases, basis bijections and source hashes are in the individual model certificates; the abbreviated names in the table are not the complete construction data.

4 Coherent model constructions

4.1 Established models and explicit coherent actions

The model certificate in `known_models/selected_models.json` gives 25 unitary realizations, with an exact unital basis bijection and all $11^3 = 1331$ structure constants checked for each. It proves existence of one model per ring, not uniqueness of associators. The standard families are pointed and representation categories, Tambara–Yamagami categories, $SU(2)_{10}$ and its indicated even counterparts, $SO(15)_2$, $SO(8)_2$, and coherent equivariantizations and associative zestings of these categories. The complete formulas, primary references, and scope of the finite searches are in `known_models/THEORY.md`.

The rank equations determine all members of the pointed, TY, A_1 , odd-metaplectic, full-even-metaplectic and half-even-metaplectic families at this rank. Prime rank prevents a nontrivial Deligne product. For strict C_2 actions on pointed categories, the rank formula $(|G| + 3|G^{C_2}|)/2 = 11$ leaves $(|G|, |G^{C_2}|) = (19, 1)$ or $(16, 2)$. The latter family is checked for all fourteen groups of order sixteen, and gives rings 1 and 112. The twisted pointed actions below are checked separately; the strict scan is not mistaken for a classification of all group-theoretical models.

For every TY equivariantization used, the certificate records a nondegenerate symmetric bicharacter and an automorphism preserving it. The automorphism therefore preserves the standard TY associator, giving an actual tensor action. The two choices of multiplication coherence are the identity and the natural parity automorphism, equal to -1 on the exceptional object and $+1$ on group objects. Exact Fourier projection computes the resulting fusion coefficients. This covers 10, 37, 64, 72, 75, 113, 157, 176. In particular a based automorphism is never used as an unverified categorical action.

A uniform zesting construction. In $SO(8)_2$, the four neutral invertibles form a Tannakian subcategory $\text{Rep}(V_4)$. Fix a coherent gauge in which their associator and mutual braiding are trivial. With respect to the V_4 grading, every bilinear map $\lambda : V_4 \times V_4 \rightarrow V_4$ is an invertible-object 2-cocycle. Its scalar associative-zesting obstruction is identically one, since it involves only the displayed associator and mutual braiding. Thus

$$X \star Y = X \otimes Y \otimes \lambda(|X|, |Y|)$$

has a unitary coherent associator. The 4^4 bilinear choices produce four based rings, 30, 57, 105, 150. The last is additional to the single- C_2 trials. The code checks all products, but the categorical existence follows from this obstruction calculation, not merely from associativity of the computed ring. Ordinary C_2 zesting also supplies ring 103 from $SU(2)_{10}$; the scalar obstruction vanishes because $H^4(C_2, U(1)) = 0$.

The spin-current pair 24, 68. Let $\mathcal{C} = SU(2)_6$ and let

$$\sigma(a) = \begin{cases} a & a \text{ even,} \\ 6 - a & a \text{ odd.} \end{cases}$$

This dimension-preserving based involution is not accepted by itself. Indeed the naive all-positive tensorator gives nonassociative proposed fusion coefficients. We use the standard real unitary quantum-Racah associator to construct its tensorator explicitly.

Put $u = 2 \cos(\pi/8)$, so $u^4 - 4u^2 + 2 = 0$ and $1847759/10^6 < u < 1847760/10^6$. The quantum integers satisfy $[1] = 1$, $[2] = u$, $[j+1] = u[j] - [j-1]$. The square of each quantum-Racah F entry lies in $\mathbb{Q}(u)$. The verifier compares all transformed squares exactly and determines

every nonzero sign by rational interval arithmetic in this fixed real embedding. At this level the usual Racah sum has no vanishing denominator; terms whose numerator contains [8] vanish. The explicit formula and checks are included in the source rather than delegated to numerical eigenvalue calculations.

There are 1664 nonzero and 16 zero admissible entries. The nonzero ratios define a consistent linear system of rank 65 in 71 normalized $\{\pm 1\}$ tensorator coefficients. The recorded solution satisfies every monoidal-functor equation. The two recorded natural isomorphisms $\mu : \sigma^2 \Rightarrow \text{Id}$ are

$$\mu_a = 1, \quad \mu_a = (-1)^a.$$

Both satisfy $\delta\mu = J\sigma(J)$ and $\mu_{\sigma(a)} = \mu_a$, including the group-associativity coherence. They therefore define two unitary C_2 actions. Orbit/stabilizer projection computes their equivariantizations: on a fixed-output fusion space the multiplicity is $(\dim + \text{tr})/2$, with trace computed from the actual tensorator and stabilizer eigenvalues. The first action yields ring 68, and the second ring 24, with every coefficient matched. The full tensorator and both coherences are part of the portable certificate.

Twisted cyclic actions giving 48, 146. Let $n = 16$, $t = 1$, and write $c(a, b) = \lfloor (a + b)/n \rfloor$ for residues in $\{0, \dots, n - 1\}$. Give the pointed category on C_n the associator

$$\omega_t(a, b, c) = \zeta_n^{tac(b, c)}.$$

For $u^2 \equiv 1 \pmod{n}$, set $p(a) = ua \bmod n$ and $v = (u^2 - 1)/n$. A tensorator $F(a) \otimes F(b) \rightarrow F(a + b)$ is

$$J(a, b) = \zeta_n^{-tua \lfloor ub/n \rfloor}.$$

The carry identity gives $\delta J = \omega_t/(p^* \omega_t)$ and shows that the exponent of $J(a, b)J(p(a), p(b))$ is $-tvab$ modulo n . Define

$$\mu_a = \zeta_{2n}^{tva^2 + 2ka}.$$

Since n is even, its coboundary is the composite tensorator. The remaining action equation is $\mu_{p(a)} = \mu_a$. It holds with $(u, v, k) = (15, 14, 0)$ and $(7, 3, 4)$, respectively.

These actions have fixed points 0 and 8, with $\mu_8 = 1$ and $J(8, 8) = -1$. The lift of 8 thus squares to the nontrivial stabilizer character, giving a cyclic group of four invertibles. The two categories are identified with rings 48 and 146. The verifier checks, for each action, 16^4 cocycle equations, 16^3 tensorator equations, 16^2 square-coherence equations, and 16 action-associativity equations, then all 1331 fusion coefficients. Every phase is a root of unity, so both constructions are unitary.

The relevant standard results are Tambara–Yamagami’s existence theorem; Gustafson–Rowell–Ruan, *Metaplectic Categories, Gauging and Property F*, Sections 3.1 and 3.3; Delaney–Galindo–Plavnik–Rowell–Zhang, *Braided zesting and its applications*; Burciu–Natale, *Fusion rules of equivariantizations of fusion categories*, corrected version 2; and the Kirillov–Reshetikhin quantum- $6j$ formula. The simple-current autoequivalence also appears in Edie-Michell’s classification of autoequivalences. None of these references is used to bypass the explicit categorical action coherence established above.

4.2 An explicit nonsplit group-theoretical model

For ring *R11_1_104* we construct an actual unitary group-theoretical category. Put $A = \mathbb{F}_2^3$, $z = (1, 0, 0)$ and

$$\sigma(a_0, a_1, a_2) = (a_0 + a_1, a_1 + a_2, a_2).$$

Let G be the order-32 group on pairs (a, e) , $0 \leq e < 4$, with

$$(a, e)(b, f) = (a + \sigma^e(b) + z\lfloor(e+f)/4\rfloor, e+f \pmod{4}).$$

Thus a lift t of the cyclic generator satisfies $t^4 = z$. Let $H = \{((0, a_1, a_2), 0)\} \cong V_4$.

The file `known_models/jordan_gt_model.json` specifies a normalized cocycle $\omega : G^3 \rightarrow \mu_8$ by its full exponent array. All 32^4 cocycle equations, all normalization equations, and $\omega|_{H^3} = 1$ are checked exactly. Consequently the category of $\mathbb{C}[H]$ -bimodules in Vec_G^ω is a unitary fusion category $\mathcal{C}(G, \omega, H, 1)$ of dimension 32. Its restriction to A is the explicit cocycle

$$\omega(a, b, c) = (-1)^{a_0(b_0c_0+b_1c_1+b_2c_2+b_1c_2)}.$$

The full relative tensor product is computed with the actual cocycle phases, by projective characters of $H \cap gHg^{-1}$. The derivation from left and right bimodule actions is given in `known_models/PROOF_JORDAN32.md`; the primary replay is `known_models/jordan_verify.py` and an independent tensor implementation is provided in `nonintegral/audit104`. The model has eleven simples, dimensions $(1^4, 2^7)$, and all 1331 fusion coefficients match $R11_1_104$ under the basis map

$$(0, 4, 8, 5, 9, 1, 2, 3, 10, 6, 7).$$

Existence rests on the verified cocycle and bimodule construction, not on associativity of a guessed fusion table. This proves one categorification, without asserting uniqueness of its associator.

5 Dimension characters, central restrictions, and a quaternionic model

Throughout this section, R_n abbreviates the supplied based ring `R11_1_n`. The integer n is its physical line number in the rank-eleven multiplicity-one input, and basis indices run from 0 to 10, with 0 the unit. All statements concern the entire based multiplication tensor. In particular, an obstruction to a complex categorification is distinguished from an obstruction that initially assumes a spherical or pseudo-unitary structure.

The results below exclude thirty-eight rings as Grothendieck rings of complex fusion categories. They also construct a unitary categorification of R_{179} , of dimension 72. The categorical ingredients are established results; the uniform applications, explicit witnesses, and finite input identifications are recorded here with exact replay certificates. These results do not assert a classification of associators on any realizable based ring.

5.1 Exact passage from arbitrary categorifications to spherical ones

We first explain the scope of the computational reduction used below. Sphericalize a hypothetical complex fusion category $\mathcal{C}$, and choose a lift of each simple object, with the unit and duality normalized. Write ε for the central sign object. In multiplicity one, the quotient of the sphericalization's Grothendieck ring by $[\varepsilon] + 1$ has structure constants

$$T_{ij}^k = \sigma_{ij}^k N_{ij}^k, \quad \sigma_{ij}^k \in \{1, -1\} \quad \text{when } N_{ij}^k = 1.$$

These constants satisfy associativity, the unit identities, cyclic trace symmetry, and reversal with duals. The trace triple (i, j, k^*) is identified only under cyclic rotation and $(a, b, c) \mapsto (c^*, b^*, a^*)$; arbitrary permutations, or commutativity of the signed quotient, are not imposed. Changing lifts replaces T_{ij}^k by $s_i s_j s_k T_{ij}^k$, where $s_0 = 1$ and $s_i = s_{i^*} \in \{1, -1\}$.

Completeness of the sign calculation. Put $\sigma = (-1)^x$ on each oriented orbit. Every associativity identity becomes an exact equation

$$\sum_m c_m (-1)^{\langle m, x \rangle} = 0.$$

If one coefficient has absolute value equal to the sum of the absolute values of the others, the positive solution and equality in the triangle inequality force the participating sign monomials to agree. Each resulting linear relation over $\mathbb{F}_2$ is replayed from its full associativity identity. An independently checked gauge matrix proves that the chosen normalization meets every gauge orbit. All remaining assignments are then enumerated and checked.

For the fifty-nine inputs in the recorded nonintegral audit, at most one binary variable remains after this reduction. The complete domain has seventy-one assignments: fifty-nine positive original tables and twelve alternative associative signed tables. Every alternative is excluded by its complete scalar-character inventory. Thus every categorically possible signed table is gauge equivalent to the original positive table.

For completeness, an associative signed table with these reciprocity identities is a finite-dimensional semisimple $*$ -algebra in its regular representation. The common kernel of its regular commutators is the sum of the one-dimensional representation spaces. Proposed scalar characters are checked directly on every product. Their distinct irreducible separating polynomials, whose degrees sum to the dimension of this common kernel, prove that no scalar character is omitted. A spherical dimension character must be nonzero on every simple and equal on dual pairs. The remaining exact obstructions use the algebraic integrality of δ/f_E and the necessary containment $f_E \in \mathbb{Q}(d_i)$; see [2, Theorem 2.13 and Corollary 2.15]. For a signed quotient these conditions are applied in the actual sphericalization: its dimension is 2δ , its two algebra summands are the original and signed rings, and their formal codegrees are doubled. Consequently the same field-containment conditions apply to both summands.

The signed conclusion has a categorical consequence, rather than merely a numerical one. Choose the lifts for which every nonzero coefficient is positive. Their products contain only chosen lifts, so they span a fusion subcategory of the sphericalization. Forgetting the lifts gives a tensor equivalence with $\mathcal{C}$. Its spherical structure therefore descends to $\mathcal{C}$; compare [2, Section 5.1].

Original dimension characters. The same complete-inventory check is performed on the fifty-nine original rings. Actual formal codegrees, rather than uncorrected regular Casimir eigenvalues, are used: on a matrix block belonging to E , the latter eigenvalue is $(\dim E)f_E$. The scalar part is extracted first and the remaining block sizes are checked by the dimension of the semisimple algebra. Exact norm and field-containment certificates then reject the inadmissible dimension characters.

For $R_2, R_{51}, R_{124}, R_{147}$ no spherical dimension character remains. For all surviving characters except those of

$$R_{36}, R_{86}, R_{95}, R_{135}, R_{136}, R_{170},$$

the inventory supplies a field embedding and a tensor sign character making every simple dimension positive. Galois conjugation of the algebraic categorical data and twisting the spherical pivotal structure by that sign character then give positive dimensions, necessarily equal to Frobenius–Perron dimensions. Thus a Galois conjugate is pseudo-unitary; see [1, Section 8.4]. This assertion does not claim that the original category is pseudo-unitary. The six exceptions have an additional mixed spherical dimension character and will be treated directly.

The exact replay checks fifty-nine sign records, twelve alternative signed character records, and fifty-nine original character records, with each record bound to the full input tensor hash. It reconstructs the sign deductions independently. The legacy sign checker also recomputes the

twelve small alternative character calculations; their results are then checked separately by direct homomorphism identities and completeness, without trusting that character-discovery calculation. This full replay, not a stored Boolean conclusion, is a dependency of the general-complex exclusions below.

5.2 The spherical dimension lattice

Let d_i be the simple dimensions of a spherical fusion category, $\delta = \sum_i d_i d_{i^*}$ its global dimension, and

$$L_d = \sum_i \mathbb{Z} d_i$$

the additive lattice generated by its simple dimensions. For an irreducible representation E of its based ring with formal codegree f_E , the induction theorem [2, Theorem 2.13] supplies a simple $A_E \in \mathcal{Z}(\mathcal{C})$ such that

$$\dim A_E = \frac{\delta}{f_E}, \quad [F(A_E) : \mathbf{1}] = \dim E, \quad (1)$$

where F is the forgetful functor. Since F preserves dimension,

$$\frac{\delta}{f_E} \in L_d \quad \text{for every } E. \quad (2)$$

This condition holds for every spherical dimension character, including characters with negative dimensions or rational global dimension.

An exact certificate for failure of (2) is a rational linear functional ℓ on the character field satisfying

$$\ell(d_i) \in \mathbb{Z} \quad \text{for every } i, \quad \ell(\delta/f_E) \notin \mathbb{Z}.$$

The verifier evaluates these expressions directly; it does not require Hermite normal form or a lattice search. Each irreducible character-field factor covers its entire Galois orbit, since both the dimension condition and the multiset of formal codegrees are Galois invariant.

Here are all thirty-four inputs rejected by the complete spherical inventory together with these lattice witnesses. Superscripts in the dimension types denote multiplicities.

1. Type $1^4, 2^3, (1 + \sqrt{5})^4$:

$$2, 31, 51, 59, 69, 83, 107, 108, 122, 124, 147, 151.$$

The Frobenius–Perron branch has $D = 40 + 8\sqrt{5}$ and $L = \mathbb{Z} + \mathbb{Z}\sqrt{5}$. The actual formal codegree $40 - 8\sqrt{5}$ requires

$$\frac{D}{40 - 8\sqrt{5}} = \frac{3 + \sqrt{5}}{2} \notin L.$$

The four inputs 2, 51, 124, 147 were already excluded by their complete spherical character inventories. For the other eight, all surviving branches are covered by exact separating functionals.

2. Type $1^4, 2^3, (2 + 2\sqrt{2})^4$:

$$32, 62, 88, 110, 137, 138, 152, 154.$$

Here $D = 64 + 32\sqrt{2}$ and $L = \mathbb{Z} + 2\mathbb{Z}\sqrt{2}$. The formal codegree 32 requires

$$\frac{D}{32} = 2 + \sqrt{2} \notin L.$$

3. Type $1^4, 2, a^2, b^2, c^2$:

$$34, 70, 89, 96, 115, 123, 139, 167.$$

Let a be the largest real root of $a^3 - 12a - 8 = 0$, and put $b = a^2/2 - 2$ and $c = a + 2$. The numbers $1, a, b$ are rationally independent, and

$$L = \mathbb{Z} + \mathbb{Z}a + \mathbb{Z}b, \quad D = 48 + 12a + 12b.$$

The formal codegree 24 requires

$$\frac{D}{24} = 2 + \frac{a+b}{2} \notin L.$$

4. Type $1^4, 2, (1 + \sqrt{5})^4, (3 + \sqrt{5})^2$:

$$36, 86, 95, 135, 136, 170.$$

The Frobenius–Perron branch has $D = 60 + 20\sqrt{5}$ and $L = \mathbb{Z} + \mathbb{Z}\sqrt{5}$; the formal codegree 40 requires $(3 + \sqrt{5})/2 \notin L$. For each of these six rings there is also a mixed spherical character of global dimension $\delta = 40$. Its simple dimensions still generate $\mathbb{Z} + \mathbb{Z}\sqrt{5}$. The actual formal codegrees $60 \pm 20\sqrt{5}$ now require

$$\frac{40}{60 \pm 20\sqrt{5}} = \frac{3 \mp \sqrt{5}}{2} \notin \mathbb{Z} + \mathbb{Z}\sqrt{5}.$$

Thus these branches are excluded directly, without an unjustified pseudo-unitary reduction.

The finite replay checks forty-four separating functionals over all remaining character branches. Four of the thirty-four inputs have no remaining spherical branch; the other thirty require at least one lattice witness. Combined with Section 5.1, these are exclusions of arbitrary complex categorifications.

5.3 A two-character central-restriction obstruction

The four further rings

$$R_3, \quad R_{52}, \quad R_{142}, \quad R_{149}$$

have dimension type $1^4, 2^3, x^4$, where $x = 2 + 2\sqrt{2}$, and global Frobenius–Perron dimension $D = 64 + 32\sqrt{2}$. For these four, the complete signed and character audit reduces existence of a complex categorification to existence of a pseudo-unitary one. We therefore work with positive simple dimensions.

Let χ be the Galois conjugate of the Frobenius–Perron character. It has values 1 on the invertibles, 2 on the two-dimensional simples, and $2 - 2\sqrt{2}$ on the four x -dimensional simples. Its formal codegree is $f_\chi = 64 - 32\sqrt{2}$. A second scalar character φ has formal codegree $f_\varphi = 16$ and takes one of the values $\pm\sqrt{2}$ on each x -dimensional simple. In input basis order, complete witnesses for this second character are as follows, with $q = \sqrt{2}$:

Input	x -dimensional indices	$(\varphi_0, \dots, \varphi_{10})$
3	4, 5, 6, 7	$(1, 1, 1, 0, -q, q, q, -q, 0, 1, -2)$
52	4, 5, 7, 8	$(1, 1, 1, 0, -q, q, -2, q, -q, 0, 1)$
142	1, 2, 3, 4	$(1, q, q, -q, -q, -2, 1, 0, 1, 1, 0)$
149	3, 4, 5, 6	$(1, 1, 1, q, q, -q, -q, 0, -2, 0, 1)$

Every product identity and both formal codegrees are checked exactly.

Write

$$\beta_\chi = \frac{D}{f_\chi} = 3 + 2\sqrt{2} = 1 + x, \quad \beta_\varphi = \frac{D}{f_\varphi} = 4 + 2\sqrt{2} = 2 + x.$$

Since both characters are one-dimensional, (1) says that the restrictions of their associated central simples contain the unit exactly once. Comparing coefficients of $\sqrt{2}$ and then rational parts, using positivity, forces

$$F(A_\chi) = \mathbf{1} + X, \quad F(A_\varphi) = \mathbf{1} + g + Y, \quad (3)$$

where X, Y have dimension x and g is a nontrivial invertible. Indeed the irrational coefficient permits exactly one x -dimensional summand, and the remaining rational weight after the prescribed unit is respectively zero or one.

The correspondence between based-ring characters and central simples, together with symmetry of the center's S -matrix, gives

$$\beta_\chi \chi(F(A_\varphi)) = \beta_\varphi \varphi(F(A_\chi)); \quad (4)$$

see [2, Example 2.9 and the discussion preceding Theorem 2.13]. But

$$\chi(F(A_\varphi)) = 4 - 2\sqrt{2}, \quad (3 + 2\sqrt{2})(4 - 2\sqrt{2}) = 4 + 2\sqrt{2}.$$

Thus (4) forces $\varphi(\mathbf{1} + X) = 1$, hence $\varphi(X) = 0$. This contradicts the displayed character values on every possible X . It excludes all four rings by the same argument, without enumerating full induction matrices. Together with the preceding thirty-four, the number of distinct general-complex exclusions in this section is thirty-eight.

5.4 A coherent quaternionic model for R_{179}

We now give an existence proof for an integral input. Let $\mathcal{D}$ be one of Izumi's unitary near-group categories for the quaternion group $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$. Its fusion rules are

$$g\rho = \rho g = \rho, \quad \rho^2 = \sum_{g \in Q_8} g + 2\rho. \quad (5)$$

Thus $\text{FPdim}(\rho) = 4$ and $\text{FPdim}(\mathcal{D}) = 24$. Existence, with the explicit categorical data used below, follows from [3, Theorems 1.1 and 6.1, Lemma 6.13].

The full integral categorical tuple. Use the faithful quaternionic representation on $K = \mathbb{C}^2$ given by

$$I = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad K_0 = IJ,$$

so that $\pi(i) = I$, $\pi(j) = J$, and $\pi(k) = K_0$. Let $j_0 = J \circ \text{complex conjugation}$. For any third root of unity ζ , Izumi's admissible tuple is

$$(K, V, U_K, j_1, j_2, \chi, l) = (\mathbb{C}^2, \pi, \pi, j_0, -\zeta j_0, \chi, 0), \quad \pi(g)\pi(h) = \chi_h(g)\pi(h)\pi(g). \quad (6)$$

This is the integral tuple of Lemma 6.13, not an application of an automorphism theorem restricted to irrational near-group categories.

Put

$$W = -\frac{1 + I + J + K_0}{2}. \quad (7)$$

Direct Gaussian-rational matrix calculations give

$$W^*W = 1, \quad W^3 = 1, \quad \text{tr}(W) = -1, \quad Wj_0 = j_0W.$$

Conjugation by W cyclically permutes I, J, K_0 . Hence, for the order-three group automorphism $\phi : i \mapsto j \mapsto k \mapsto i$,

$$W\pi(g)W^* = \pi(\phi(g)) \quad (g \in Q_8).$$

These equations preserve both representations V and U_K in (6), both antiunitaries j_1, j_2 , and the commutator bicharacter χ after relabeling by ϕ . Since $l = 0$, its compatibility is automatic. This verifies every component of the equivalence-of-tuples condition in [3, Definition 4.8].

Categorical action and coherence. For clarity, existence of the action can be read directly from the reconstruction, not merely from a permutation of fusion rules. The generating isometries are $S_g \in \text{Hom}(g, \rho^2)$ and $T(v) \in \text{Hom}(\rho, \rho^2)$, $v \in K$. Define the Cuntz-algebra automorphism

$$\gamma(S_g) = S_{\phi(g)}, \quad \gamma(T(v)) = T(Wv). \quad (8)$$

The reconstruction equations [3, (3.10)–(3.12)] are covariant under this assignment. In (3.10), $\sum_g S_g$ and the contraction $\sum_a T_a j_1(T_a)$ are preserved. Equation (3.11) is covariant because the pointed representations intertwine. Each term in (3.12) is covariant because W intertwines V, U_K , commutes with j_1, j_2 , and preserves $l = 0$. Therefore γ conjugates the pointed endomorphisms by ϕ and commutes with ρ . The reconstruction theorem [3, Theorem 4.9] supplies the resulting unitary tensor autoequivalence of $\mathcal{D}$.

All morphisms are generated by these isometries and their adjoints [3, Remark 3.6]. Since $\phi^3 = 1$ and $W^3 = 1$, the cube of (8) is the identity on the Cuntz generators, hence on the reconstructed category. Identity compositors consequently give a coherent, strict C_3 action. No use is made of Izumi's Section 13 automorphism results, which have a different, irrational scope.

Equivariant simples and products. The unitary equivariantization $\mathcal{C} = \mathcal{D}^{C_3}$ has dimension 72. The fixed invertibles $1, -1$ each have three equivariant structures, giving six invertibles $L_{(g,a)}$, $g \in \{1, -1\}$, $a \in \mathbb{Z}/3$. The free orbits $\{i, j, k\}$ and $\{-i, -j, -k\}$ give two objects U, V of dimension three. The fixed object ρ has three lifts P_a of dimension four. The rank and dimension type are therefore 11 and $1^6, 3^2, 4^3$.

The pointed part is computed by equivariant homogeneous fibers. Explicitly,

$$\begin{aligned} L_{(g,a)}L_{(h,b)} &= L_{(gh,a+b)}, \\ L_{(1,a)}U &= UL_{(1,a)} = U, & L_{(1,a)}V &= VL_{(1,a)} = V, \\ L_{(-1,a)}U &= UL_{(-1,a)} = V, & L_{(-1,a)}V &= VL_{(-1,a)} = U, \\ U^2 = V^2 &= \sum_{a \in \mathbb{Z}/3} L_{(-1,a)} + U + V, & UV = VU &= \sum_{a \in \mathbb{Z}/3} L_{(1,a)} + U + V. \end{aligned}$$

The remaining products are

$$L_{(g,a)}P_b = P_bL_{(g,a)} = P_{a+b}, \quad (9)$$

$$UP_b = P_bU = VP_b = P_bV = P_0 + P_1 + P_2, \quad (10)$$

$$\begin{aligned} P_aP_b &= L_{(1,a+b)} + L_{(-1,a+b)} + U + V \\ &\quad + P_{a+b+1} + P_{a+b+2}. \end{aligned} \quad (11)$$

Indices are read modulo three. In (11), the fixed pointed channels have phase one because S_1, S_{-1} are fixed by γ . The action on $\text{Hom}(\rho, \rho^2)$ is W , whose eigencharacters are the two nontrivial

characters of C_3 . An inverse convention for the action replaces W by W^{-1} and leaves this multiset unchanged. In the right absorption channels of (9), no additional phase occurs: $\pi(\pm 1)$ commutes with W . Free three-cycles contribute each character once, yielding (10). Finally $P_a^* = P_{-a}$, so exactly one four-dimensional simple is self-dual.

In the model basis

$$L_{(1,0)}, L_{(1,1)}, L_{(1,2)}, L_{(-1,0)}, L_{(-1,1)}, L_{(-1,2)}, U, V, P_0, P_1, P_2,$$

the model-to-input bijection is

$$(0, 7, 8, 9, 1, 2, 5, 6, 10, 4, 3). \quad (12)$$

Every one of the $11^3 = 1331$ multiplication coefficients agrees under this bijection with R_{179} . Its canonical tensor SHA-256 is

a9492a728c3c2e5d59d92d723a5349782a
962bc04ead4aacd0d7756e5d7c95f2.

All three cube-root choices in (6) provide this based tensor. One such realization suffices for the existence claim.

5.5 Finite verification and dependency order

All following paths are relative to `work/rank11/nonintegral/`. Run the four accepted entry-points in this order, from the project root:

1. `verify_reduction.py`: complete signed and scalar replay; 59 sign records, 12 alternative signed character records, and 59 original character records.
2. `verify_spherical_lattice.py`: 34 excluded inputs, including the four already rejected scalar inventories, and 44 exact separating functionals.
3. `verify_two_character.py`: the four explicit character pairs, formal codegrees, forced restrictions, and their contradictions. Its rational arithmetic is implemented directly in $\mathbb{Q}(\sqrt{2})$.
4. `integral179/verify.py`: an independent reconstruction of the equivariant fusion tensor by

$$\dim H^{C_3} = \frac{1}{3} \sum_{t=0}^2 \text{Tr}(t \mid H)$$

on every underlying near-group fusion space. The exceptional two-dimensional space has character $(2, -1, -1)$; the other channels are the one-dimensional absorption or S_g spaces, with their orbit permutations. This verifier does not import the model constructor. It checks the exact action matrix, the source hash, all 1331 tensor coefficients, and all $11^4 = 14641$ associativity identities.

The first replay imports the project's prior oriented-sign and exact character verifiers. The lattice replay imports the shared exact-number field helpers. The two-character and quaternionic model verifiers use only the Python standard library. Critical conditions use explicit exceptions and remain active under `python -O`. In particular, the second and third checks alone are not certificates of a general-complex exclusion: they require the full first replay, not merely its summary flags. No timeout, failed model search, numerical tolerance, or incomplete finite domain is used in any accepted conclusion here.

6 A central dihedral obstruction in dimension 120

Theorem. The based rings $R_{11_1_40}$ and $R_{11_1_82}$ admit no categorification by a fusion category over $\mathbb{C}$.

Exact hypotheses. Both rings are commutative of rank eleven, with dimensions $1^2, 2, 3^2, 4^6$. Their nontrivial invertible g has order two; its multiplication permutation exchanges the two invertibles, exchanges the two dimension-three simples, and fixes the other seven simples. The basis elements of dimension less than four form a closed subring of dimension twenty-four. No identification of this subring with a particular category is assumed: in the second input its two dimension-three simples are dual to each other, unlike the representation ring of S_4 .

For a scalar character χ , put $f_\chi = \sum_i \chi(b_i) \chi(b_i^*)$ and $\beta_\chi = 120/f_\chi$. The eleven values β_χ are

$$1, 2, 2, 8, 8, 8, 8, 8, 15, 30, 30.$$

Both characters with $\beta_\chi = 2$ satisfy $\chi(g) = 1$. Finally the induction Gram coefficient B_{gg} is eleven. The accompanying exact verifier checks these statements directly from the source tensor and a complete character table, including every character multiplication identity. No restriction-frame search is used.

A forced central dihedral category. Suppose that $\mathcal{C}$ is a categorification. Its integrality supplies the canonical positive spherical structure by [1, Proposition 8.24]. Let $F : Z(\mathcal{C}) \rightarrow \mathcal{C}$ be forgetful, with right adjoint I . By [2, Theorems 2.5 and 2.13], the simple constituents A_χ of $I(\mathbf{1})$ occur once, have $\dim(A_\chi) = \beta_\chi$, satisfy $[F(A_\chi) : \mathbf{1}] = 1$, and have trivial twist. The character identification preceding Theorem 2.13 gives

$$S_{A_\chi, Y} = \beta_\chi \chi(F(Y)) \quad (Y \text{ simple in } Z(\mathcal{C})),$$

where S is unnormalized. For the two dimension-two constituents A, B , positivity forces $F(A) = F(B) = \mathbf{1} \oplus g$, whence

$$S_{A,A} = S_{A,B} = S_{B,B} = 4.$$

These equalities force trivial monodromy: the balancing formula is a positive weighted sum of roots of unity, and equality with the sum of the positive weights forces every eigenvalue to be one. Therefore the subcategory $\mathcal{T} = \langle A, B \rangle$ is symmetric and has trivial twist. It is Tannakian; see also [5].

All objects of $\mathcal{T}$ forget to sums of $\mathbf{1}$ and g . If a simple central Y with this property contains $\mathbf{1}$ after forgetting, it lies in $I(\mathbf{1})$, with unit multiplicity one. Write $F(Y) = \mathbf{1} \oplus mg$. The induction Gram equality gives $m^2 \leq B_{gg} = 11$, so $\dim(Y) \leq 4$. The displayed β list therefore leaves only $Y = \mathbf{1}, A, B$. In particular, the only simple central object supported solely on $\mathbf{1}$ is $\mathbf{1}$ itself.

If instead $F(Y) = mg$, then $F(Y^*Y) = m^2 \mathbf{1}$; all constituents of Y^*Y must be the central unit. Simplicity of Y gives $m^2 = 1$. Such a Y is invertible and forgets to g . There is at most one such lift, since the ratio of two lifts forgets to the unit; call it z if it exists. It must occur in $\mathcal{T}$: the dimension-four object A^*A contains the unit exactly once, and its remaining dimension three cannot be filled using only the dimension-two simples A, B . Thus $\mathcal{T}$ has exactly $\mathbf{1}, z, A, B$, of dimensions 1, 1, 2, 2. It is consequently $\text{Rep}(D_{10})$, where D_{10} has order ten. Its unique nontrivial invertible z is a central boson with $F(z) = g$.

De-equivariantization and a faithful grading. The subcategory $\mathcal{E} = \langle z \rangle \simeq \text{Rep}(C_2)$ embeds in $\mathcal{C}$ under F . By [4, Proposition 2.10], de-equivariantization gives a category $\mathcal{D}$ of dimension

sixty with $\mathcal{C} \simeq \mathcal{D}^{C_2}$. A free orbit $\{X, gX\}$ gives one simple of dimension $\dim(X)$; a fixed X gives two simples of dimension $\dim(X)/2$. Here the fixed stabilizer algebra has two one-dimensional representations, since $H^2(C_2, \mathbb{C}^\times) = 0$, and its sign automorphism interchanges them. The resulting dimension list is

$$\mathcal{D} : \quad 1^3, 2^{12}, 3, \quad \text{rank}(\mathcal{D}) = 16.$$

Since $\mathcal{T}$ is symmetric, it lies in $\mathcal{E}'$. The central equivalence $\mathcal{E}'_{C_2} \simeq Z(\mathcal{D})$ sends $\mathcal{T}_{C_2}$ to $\text{Rep}(C_5)$. Every object of this subcategory forgets to a multiple of the unit: $\mathbf{1}, g$ become the unit under de-equivariantization, and the images of A, B become two copies of it. Their four nontrivial C_5 -character summands must each forget to the unit by positivity. Proposition 2.9(i) of [4] now supplies a faithful C_5 grading of $\mathcal{D}$, whose components all have dimension twelve.

The closed dimension-twenty-four subcategory above de-equivariantizes to a subcategory $\mathcal{K} \subset \mathcal{D}$ of dimensions 1, 1, 1, 3. Its invertibles form $C_3 = \langle a \rangle$, and its unique other simple X is self-dual and satisfies

$$aX = Xa = X, \quad X^2 = \mathbf{1} + a + a^2 + 2X.$$

Thus $\mathcal{K}$ has the fusion ring of $\text{Rep}(A_4)$. It equals its adjoint category, so every grading restricts trivially to $\mathcal{K}$. Equality of dimensions gives $\mathcal{D}_e = \mathcal{K}$.

The Morita class of every possible neutral category. By [4, Theorem 1.6], $\mathcal{K}$ is solvable. A solvable tower consists of extensions and equivariantizations by cyclic groups of prime order [4, Definition 1.2 and Proposition 4.4]. Its final step cannot be an extension because $\mathcal{K}$ has trivial universal grading. Hence $\mathcal{K} = \mathcal{L}^{C_p}$ for a prime p . The embedded $\text{Rep}(C_p)$ forces $p = 3$, and $\text{FPdim}(\mathcal{L}) = 4$.

The simple equivariant dimensions are $[C_3 : H] \dim(\pi) \dim(Y)$. All projective irreducible representations of the cyclic stabilizers H are one-dimensional. Since the only dimensions in $\mathcal{K}$ are one and three, a fixed Y has dimension one or three, the latter impossible in a category of dimension four. A nonfixed Y has an orbit of length three and likewise has dimension one. Thus $\mathcal{L} = \text{Vec}_H^\eta$ for a group H of order four. The action on H is nontrivial, since otherwise equivariantization would have twelve invertibles. Thus $H = V_4$, with C_3 cycling its nonidentity elements.

The pointed crossed product $\mathcal{L} \rtimes C_3$ is Morita equivalent to $\mathcal{K}$ by [4, Proposition 2.2]. Its group of invertibles is the extension of V_4 by C_3 with this nontrivial action; the coprime extension splits, giving A_4 . Consequently

$$\mathcal{K} \text{ is Morita equivalent to } \text{Vec}_{A_4}^\Omega$$

for some three-cocycle Ω , with no triviality assumption on Ω .

No order five in the twisted Brauer–Picard group. Let $\mathcal{P} = \text{Vec}_{A_4}^\Omega$. The twisted classification [6, Theorem 1.2, or Theorem 4.12] parametrizes Lagrangian subcategories of $Z(\mathcal{P})$ by normal abelian subgroups and invariant alternating Ω -bicharacters. The only possible subgroups are $1, V_4$. The first contributes one choice; for V_4 , the ratio of two admissible twisted bicharacters is an ordinary alternating bicharacter, of which there are exactly two. Hence there are at most three Lagrangian subcategories.

The stabilizer of the canonical $\text{Rep}(A_4)$ preserves its regular algebra $\text{Fun}(A_4)$: a connected étale algebra of dimension twelve in $\text{Rep}(A_4)$ corresponds uniquely to the free transitive A_4 -set. Conversely the regular algebra generates that subcategory. Theorem 3.3 and Remark 3.4 of [7] identify this stabilizer with the induction image $I(A_4, \Omega)$. The elementary bound from tensor autoequivalences already suffices: their kernel over the underlying automorphism group is

$H^2(A_4, \mathbb{C}^\times) = C_2$ and their image is contained in $\text{Aut}(A_4) = S_4$, so the stabilizer order divides 48.

More precisely, Proposition 4.5 of [7], valid for arbitrary Ω , gives

$$Z(A_4) \longrightarrow H^2(A_4, \mathbb{C}^\times) \longrightarrow I(A_4, \Omega) \longrightarrow \text{OutStab}(\Omega) \longrightarrow 1.$$

Since $Z(A_4) = 1$ and $\text{Out}(A_4) = C_2$, the stabilizer order is two or four. These group-theoretic data are recorded in [8, Section 8.3]; its untwisted Brauer–Picard computation is not being applied to a twisted category. Orbit–stabilizer, with orbit size at most three, shows that $\text{BrPic}(\mathcal{P}) \simeq \text{Aut}_{\text{br}}(Z(\mathcal{P}))$ has no element of order five. Morita invariance gives the same statement for $\mathcal{K}$.

Contradiction. A faithful graded extension determines a homomorphism

$$C_5 \longrightarrow \text{BrPic}(\mathcal{K}), \quad h \longmapsto [\mathcal{D}_h].$$

It is trivial, so every component is equivalent to the regular $\mathcal{K}$ -bimodule category and has rank four. Thus $\text{rank}(\mathcal{D}) = 20$, contrary to the forced rank sixteen. This proves the theorem for both inputs. $\square$

7 A cyclic extension restriction obstruction

Theorem. The based ring R11_1_153 admits no categorification over $\mathbb{C}$. The argument applies to every complex fusion category, without a braiding, pivotality, or unitarity assumption.

A general restriction criterion. Let $\mathcal{D}$ be a fusion category, G a finite group, $K \leq G$, and $c : G \rightarrow \text{BrPic}(\mathcal{D})$ a fixed homomorphism. Put $A = \text{Inv } Z(\mathcal{D})$ with the action induced by c . Suppose a reference G -extension realizing c exists and

$$\text{res} : H^2(G, A) \longrightarrow H^2(K, A) \quad \text{is zero.}$$

Then the K -subextensions of all G -extensions realizing c have isomorphic based fusion rings, respecting their gradings and neutral subcategories up to the chosen identifications.

Indeed, systems of multiplication functors form an $H^2(G, A)$ -torsor by [9, Theorem 8.5]. Comparing two existing extensions, their product systems differ by an A -valued 2-cocycle. Restricting the multiplication functors restricts this cocycle. If its restricted class is zero, componentwise bimodule equivalences remove the difference, so the restricted fusion coefficients agree. The remaining $H^3(G, \mathbb{C}^*)$ -freedom changes associators by scalars and leaves multiplication functors unchanged [9, Theorem 8.9]. Duality is determined by the coefficient of the unit, so it is preserved. This argument compares existing extensions and requires no separate obstruction-vanishing assertion.

Exact structural hypotheses. Both R11_1_104 and R11_1_153 have dimension type $(1^4, 2^7)$, universal grading C_4 , and homogeneous ranks $(5, 2, 2, 2)$. Their neutral ring is the Tambara–Yamagami ring on V_4 ; write m for its dimension-two simple. In a generating component there is a simple X with

$$XX^* = \mathbf{1} + h_L + m, \quad X^*X = \mathbf{1} + h_R + m, \quad h_L \neq h_R \neq \mathbf{1}, \quad h_L \neq \mathbf{1}.$$

The source verifier derives the adjoint subring and all homogeneous components directly from the tensors and checks every homogeneous product, rather than only products of chosen representatives.

The neutral category is necessarily $\text{Rep}(D_8)$. Every categorification of the neutral ring is $\text{TY}(V_4, \chi, \tau)$, with χ a nondegenerate symmetric bicharacter and $\tau = \pm \frac{1}{2}$. The canonical internal-end algebras XX^* and X^*X have the form $\mathbf{1} + h + m$. Their multiplication maps $h \otimes m \rightarrow m$ and $m \otimes h \rightarrow m$ are nonzero: the h -summand arises from an invertible stabilizing action on X . Associativity on (h, m, h) therefore forces $\chi(h, h) = 1$. There are two distinct nonzero isotropic elements, h_L, h_R . A nonalternating nondegenerate symmetric form on $\mathbb{F}_2^2$ has exactly one, so χ is alternating. The two remaining TY categories are $\text{Rep}(D_8)$ and $\text{Rep}(Q_8)$, with groups of order eight. But $\text{BrPic}(\text{Rep}(Q_8)) \cong S_3$ [8, Remark 9.4]. Any homomorphism from C_4 to this group kills its degree-two element, forcing that homogeneous bimodule to be regular of rank five, contrary to its rank two. Thus $\mathcal{D} \simeq \text{Rep}(D_8)$. This includes all neutral associators and both TY signs.

The Brauer–Picard homomorphism is unique up to neutral equivalence. By [8, Section 9.1], $\text{BrPic}(\mathcal{D}) \cong S_4$. Reindexing the alternating TY data realizes the six isometries of V_4 as tensor autoequivalences. Their map into the Brauer–Picard group is injective: its kernel consists of inner tensor autoequivalences, which are tensor isomorphic to the identity in a braided category. This S_3 is the stabilizer of the regular left-module class, a point p of the four-point S_4 action. All three nonidentity components have rank two, so the first, second, and third powers of the grading generator move p . Its image is therefore a 4-cycle. The S_3 fixing p conjugates the six 4-cycles transitively: its intersection with the centralizer of one is trivial. Consequently every relevant c can be aligned with the reference by an actual tensor autoequivalence of $\mathcal{D}$.

The coefficient module. Here $A = \text{Inv } Z(\mathcal{D}) \cong C_2^3$ and, as an S_4 -module,

$$A \cong \mathbb{F}_2^4 / \langle (1, 1, 1, 1) \rangle.$$

For completeness this action can be identified intrinsically. Let z be the nontrivial central invertible with underlying tensor unit: its half-braiding acts as $+1$ on the four invertibles of $\mathcal{D}$ and -1 on m . Every tensor autoequivalence fixes z , so its S_4 orbit has size one or four. If its orbit had size one, the C_4 action of the explicit `R11_1_104` model would fix z . Relative-center equivariantization [10] would then lift z to a central invertible with underlying unit: the projective linearization obstruction lies in $H^2(C_4, \mathbb{C}^*) = 0$. But tensor automorphisms of the identity of that category are characters of its universal grading C_4 , hence restrict trivially to its adjoint category $\mathcal{D}$. This contradicts the nontrivial restriction z . Thus the orbit consists of four distinct nonzero vectors. They span A , since a two-dimensional space over $\mathbb{F}_2$ has only three nonzero vectors. The unique relation is S_4 -invariant and is their sum zero, proving the displayed module identification.

Vanishing of the restriction. Let t be a 4-cycle and $N = t - 1$ on A . This is a single unipotent Jordan block of size three:

$$N^3 = 0, \quad \text{rank } N = 2, \quad \text{rank } N^2 = 1, \quad \ker N = \text{im } N^2.$$

The C_4 norm is $1 + t + t^2 + t^3 = N^3 = 0$, so cyclic cohomology gives

$$H^2(C_4, A) = \ker N, \quad H^2(\langle t^2 \rangle, A) = \ker N^2 / \text{im } N^2.$$

Even-degree restriction sends an invariant representative to itself. Since $\ker N = \text{im } N^2$, this restriction is zero. This computation is also verified without invoking the periodic resolution: an independent normalized bar-cochain program enumerates all 128 normalized C_4 2-cocycles, finds the 64 coboundaries, and supplies a C_2 1-cochain trivializing the restriction of every cocycle. It also verifies the six conjugacy representatives above.

Reference model and contradiction. The reference R11_1_104 is the explicit group-theoretical category $\mathcal{C}(G, \omega, H, 1)$ provided by the finite certificate `jordan_gt_model.json`. Here $|G| = 32$, $H \cong V_4$, and ω is a normalized μ_8 -valued 3-cocycle trivial on H . The group is the nonsplit Jordan extension of C_2^3 by C_4 , with t^4 equal to the fixed nonzero vector. The certificate includes the complete group table and cocycle, all simple labels, the full model fusion tensor, and a unit-preserving basis bijection to the original census tensor. All 32^4 cocycle identities and all fusion coefficients are checked. An independent implementation reconstructs all 1331 coefficients from free algebra bimodules, balanced tensor quotients, and graded intertwiner systems; it imports no projective-character convolution routine. This audit passes and is executed by the exclusion verifier.

In R11_1_104, the degree-two simples have source labels 9,10 and are both self-dual. In R11_1_153, those labels are 1,2 and they are dual to each other. Thus the respective even subrings have seven and five self-dual simples and are not isomorphic as based rings. A hypothetical categorification of the latter ring has the forced neutral category and the same 4-cycle homomorphism as the reference, up to neutral equivalence. The restriction criterion forces its even subring to equal that of the reference, a contradiction.

Scope and reproducibility. The exclusion uses the new general restriction criterion and a positive reference model, not a claim that a bounded model search is exhaustive. The earlier failed cocycle ansätze have no role in the proof. The source-bound records are `c4_reduction.json` and `c4_exclusion.json`; the independent normalized-cochain audit is `c4_torsor_independent.json`. The categorical neutral-module audit is `C4_NEUTRAL_MODULE_AUDIT.md`.

8 Complete status index

Excluded (155). 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 29, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 47, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 177, 178, 180, 181.

Unitary models (27). 1, 10, 24, 26, 28, 30, 37, 45, 46, 48, 57, 64, 68, 72, 75, 103, 104, 105, 112, 113, 146, 150, 157, 171, 176, 179, 182.

Every number above denotes the suffix in R11_1_i. The original input has uncompressed SHA-256

```
3186c410888e0bc7f30c9e423e0184a156
18246bc9f4d0ef503e708c4081eff8
```

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